Supplemental material for "Universal properties in ultracold ion-atom interactions"

Bo Gao
Department of Physics and Astronomy, University of Toledo, MS 111, Toledo, Ohio 43606

(Dated: April 29, 2010)

We give here a brief summary on the computation of the characteristic exponent of the modified Mathieu function, and an example illustrating the meaning of the universal resonance spectrum.

PACS numbers:

I. COMPUTATION OF THE CHARACTERISTIC EXPONENT OF THE MODIFIED MATHIEU FUNCTION

It is well known that one way to calculate the characteristic exponent of the modified Mathieu function is through a Hill determinant, a quantity denoted by $\Delta^l(0)$ in Refs. [15,16], and will be denoted by $\mathcal{H}_l(\epsilon_s)$ here to emphasize that it is a function of the scaled energy. The determinant can be computed directly [15], except that since it is a determinant of a matrix of infinite dimensions, one needs to be careful to ensure its convergence without loosing accuracy [16].

We have developed a different approach to the evaluation of the Hill determinant. Instead of direct evaluation, we have shown that it can be obtained from the functions introduced in this article. Specifically, we have shown that

$$\mathcal{H}_l(\epsilon_s) = \frac{1}{[C_{\epsilon_s l}(\nu=0)]^2}\left[1 + \frac{2\epsilon_s}{\nu_0^2(4-\nu_0^2)}Q(\nu=0)\right] , \tag{1}$$

in which all quantities are as defined in the article. Note that $Q(\nu=0)$ and $C_{\epsilon_s l}(\nu=0)$ are both functions of the scaled energy ϵ_s, and they can be computed easily to desired precisions.

From $\mathcal{H}_l$, the ν, as a function of energy, can be obtained in a straightforward manner. For $0 \le \mathcal{H}_l \le 2$, ν is real, and is given by

$$\nu = \begin{cases} l + \frac{1}{\pi}\cos^{-1}(1-\mathcal{H}_l) , & l = \text{even} \\ l + 1 - \frac{1}{\pi}\cos^{-1}(1-\mathcal{H}_l) , & l = \text{odd} \end{cases} , \tag{2}$$

in which $\cos^{-1}(x)$ is taken to be within $[0,\pi]$. For $\mathcal{H}_l < 0$ or $\mathcal{H}_l > 2$, $\nu = \nu_r + i\nu_i$ is complex, with its imaginary part ν_i given by [15]

$$\begin{aligned} \nu_i &= \frac{1}{\pi}\cosh^{-1}\left(|1-\mathcal{H}_l|\right) , & (3) \\ &= \frac{1}{\pi}\ln\left[|1-\mathcal{H}_l| + \sqrt{(1-\mathcal{H}_l)^2 - 1}\right] , & (4) \end{aligned}$$

and its real part ν_r given by

$$\nu_r = \begin{cases} l , & l = \text{even} \\ l+1 , & l = \text{odd} \end{cases} , \tag{5}$$

for $\mathcal{H}_l < 0$, and by

$$\nu_r = \begin{cases} l+1 , & l = \text{even} \\ l , & l = \text{odd} \end{cases} , \tag{6}$$

for $\mathcal{H}_l > 2$. Note that $\nu + 2j$, where j is an integer, is equivalent to ν.

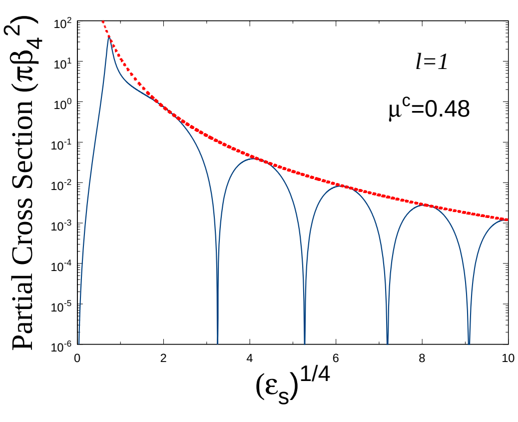

FIG. 1: The p partial scattering cross section (solid line) with $\mu^c = 0.48$. The dashed line represents the p wave unitarity limit.

II. AN EXAMPLE ILLUSTRATING THE MEANING OF THE UNIVERSAL RESONANCE SPECTRUM

Figure 1 illustrates the partial scattering cross section for a p wave with a quantum defect of $\mu^c = 0.48$. The universal resonance spectrum gives the points at which the cross section reaches its unitarity limit of $\sigma_{l=1}/(\pi\beta_4^2) = 12/\epsilon_s$. The first peak in the figure corresponds to a shape resonance. Other points at which the cross section reaches its unitarity limit correspond to positions of resonances with negative widths. They are called here diffraction resonances.