PHYSICAL REVIEW A VOLUME 41, NUMBER 9 1 MAY 1990

Multiphoton detachment in a static uniform magnetic field

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(Received 9 November 1989)

Multiphoton detachment cross sections for a negative ion in a static, uniform magnetic field are obtained in the approximation that electron-atom interactions are ignored in the final state. The final state is described by the analytic momentum-space wave function for an electron in the combined field of a laser and a static uniform magnetic field. The effects of a static-field-induced electron-photon interaction are stressed. Three consequences of our general formulas are discussed. First, the ponderomotive shift is modified by the magnetic field. It goes through a resonance at the cyclotron frequency and becomes negative for smaller frequencies. Second, for N-photon detachment of an s-shell electron by a laser of arbitrary strength, the effect of a weak magnetic field can be described near threshold by two modulation factors, one for even N and one for odd N, which depend only on a scaled energy. Third, cyclotron resonances are shown to exist under certain conditions, as demonstrated here for the specific example of multiphoton detachment of H^- by a weak laser in an arbitrarily strong magnetic field.

I. INTRODUCTION

Since the pioneering measurements by Blumberg, Jopson, and Larson,[1] the subject of photodetachment in a weak magnetic field has stimulated much theoretical discussion.[2-8] After the initial explanation by Blumberg, Itano, and Larson,[2] most efforts have been devoted to incorporating the effect of the atomic core on the detached electron. This has been achieved by Greene[8] through the use of a frame transformation technique originated by Harmin[9] and Fano[10] in their work on photoionization in an electric field. The basis of this technique is that the effect of the weak magnetic field on the photodetachment cross section comes mainly from its change of the asymptotic channels into which the electron escapes, whereas the photon absorption process, which happens in this case only at short distances, is unaffected by the magnetic field. The absence of the coupling between the electron motion in a magnetic field and the laser radiation is also implicitly assumed. As a result, the photodetachment cross section in a weak magnetic field can be factored as the product of the field-free cross section and a modulation factor. By employing the analytic solution[11] of an electron in the combined field of a laser and a static magnetic field, we show here that a different picture applies when the magnetic field becomes very strong, or more precisely, when the photon frequency is close to the cyclotron frequency.

Our interest in the present problem is stimulated by our recent work[12] on the related problem of multiphoton detachment in a static electric field using the analytic solution of an electron in the combined field of a laser and a static electric field. We had two major results. First, for N-photon detachment of an s-shell electron near the detachment threshold, the effect of a *weak electric field* can be characterized by two modulation factors, one for even N and one for odd N. This result confirms indirectly the expectation that the central idea of a frame transformation should apply to multiphoton detachment in a weak static field, since the short-range character of the electron-photon interaction does not depend either on the number of photons absorbed or on the laser intensity.[13] Second, maybe more importantly, the multiphoton detachment cross sections in a *strong electric field* differ significantly from the predictions of weak field theory due to what we call the field coupling effect, even for weak laser intensities.

A similar dichotomy between weak and strong static field effects is also found in the magnetic field case. To understand these results and the field coupling effect in particular, one has to realize that, firstly, in the absence of any static fields, the very reason for the applicability of perturbation theory, when the laser is weak, is the short-range character of the electron-photon coupling, or equivalently, the fact that a free electron cannot interact with photons. Otherwise, at large distances, the laser field is the single most dominant force and cannot be regarded as a perturbation. Secondly, an electron in a static electric or magnetic field can indeed absorb or emit photons. Keeping these in mind, it is not difficult to understand that, in the presence of the infinite-range potential such as the static electric or magnetic field, the asymptotic channels cannot generally be defined by the motion in the static field alone, since this motion is now coupled to the laser field, unless this coupling is weak compared to that induced by the atomic potential. In the case of a static uniform electric field, this condition for the separability between the effect of the external static field and that of the laser field is equivalent to requiring the external static potential to be much smaller than the atomic potential. But it is more subtle for a static uniform magnetic field because the motion in it could be resonantly coupled to the laser field at the cyclotron frequency. Hence, besides requiring the magnetic field to be much smaller than the atomic potential, we must also require it to be small enough that the cyclotron frequency is much smaller than the laser frequency. This condition is indeed satisfied in Refs. 2–8, where only single-photon detachment is considered.

The coupling between the motion in a uniform magnetic field and the laser field is included in our theory by using the analytic solution[11] of an electron in the combined fields. The main text will discuss only the most interesting case, in which the laser polarization is perpendicular to the magnetic field. The special case of parallel magnetic field and laser polarization, in which the separability of the magnetic field effect and the laser radiation effect on the electron motion is retained at all magnetic and laser field strengths, is discussed briefly in the Appendix. More details are given elsewhere.[14]

In Sec. II, we present the exact momentum-space wave function for an electron in combined laser and static uniform magnetic fields using the known solution of a forced harmonic oscillator.[15] The momentum-space representation will be used throughout this paper because of the free-electron nature of the problem. General formulas for multiphoton detachment cross sections and the modification of the ponderomotive shift by the magnetic field are derived in Sec. III by following closely the approach used by Reiss[16] for the case of multiphoton detachment in the absence of any external fields. Section IV considers the weak-magnetic-field limit, while Sec. V is devoted to the specific case of multiphoton detachment of H^- by a weak laser in an arbitrarily strong magnetic field. Conclusions and discussions are given in Sec. VI.

II. WAVE FUNCTION OF AN ELECTRON IN COMBINED LASER AND MAGNETIC FIELDS

Let the laser polarization be along the z axis and the magnetic field be along the x axis. We can choose the gauge such that the combined field is described (in the electric-dipole approximation) by

$$\mathbf{A}=(0,-Bz,(cE_0/\omega)\cos\omega t)\ . \tag{1}$$

The time-dependent Schrödinger equation for a free electron in this field, written in momentum space, is then

$$i\frac{\partial}{\partial t}\psi_f=(\tfrac{1}{2}p^2+V_f)\psi_f\ , \tag{2}$$

where

$$V_f=-i\omega_c p_y\frac{\partial}{\partial p_z}-\tfrac{1}{2}\omega_c^2\frac{\partial^2}{\partial p_z^2}+\frac{p_zE_0}{\omega}\cos\omega t+\frac{E_0^2}{2\omega^2}\cos^2\omega t\ ,$$

in which $\omega_c=B/c$ is the cyclotron frequency and B is given in atomic units $B_a=e/a_0^2=1.7153\times10^7$ G. To get a better feeling for the correspondence of the cyclotron frequency to the magnetic field strength, we can write their relation in more conventional units as ν_c (GHz)$=28B$ (T).

Equation (2) is separable in momentum coordinates, i.e.,

$$\psi_f(\mathbf{p},t)=\psi_f^x(p_x,t)\psi_f^y(p_y,t)\psi_f^z(p_z,t)\ . \tag{3}$$

Since p_x and p_y commute with the Hamiltonian and therefore are good quantum numbers, we have

$$\psi_f^x(p_x,t)=\delta(p_x-p_x^f)e^{-i\varepsilon_f^x t}\ , \tag{4}$$

$$\psi_f^y(p_y,t)=\delta(p_y-p_y^f)e^{-i\varepsilon_f^y t}\ , \tag{5}$$

where $\varepsilon_f^x=p_x^{f\,2}/2$ and $\varepsilon_f^y=p_y^{f\,2}/2$. Substituting Eqs. (3)–(5) into Eq. (2) and letting

$$\psi_f^z(p_z,t)=\exp[-i(p_y^f p_z/\omega_c-\varepsilon_f^y t+st+v\sin2\omega t)]\eta(p_z,t)\ , \tag{6}$$

where $s\equiv E_0^2/4\omega^2$ is the ponderomotive shift and $v\equiv E_0^2/8\omega^3$, we obtain the following equation for $\eta(p_z,t)$:

$$i\frac{\partial}{\partial t}\eta(p_z,t)=\left[-\tfrac{1}{2}\omega_c^2\frac{\partial^2}{\partial p_z^2}+\tfrac{1}{2}p_z^2+\frac{p_zE_0}{\omega}\cos\omega t\right]\eta(p_z,t)\ . \tag{7}$$

Equation (7) is the same as the equation for a harmonic oscillator with mass $1/\omega_c^2$ and natural frequency ω_c in an oscillating external field, which has been the subject of many investigations since the early 1950s.[15] Specifically we have

$$\eta_{n_z}(p_z,t)=\frac{\omega_c^{-1/4}}{(2^{n_z}n_z!\sqrt{\pi})^{1/2}}H_{n_z}([p_z-\xi(t)]/\omega_c^{1/2})\times\exp\left[-\frac{1}{2\omega_c}[p_z-\xi(t)]^2-i\frac{E_0}{\omega^2-\omega_c^2}\sin\omega t[p_z-\xi(t)]-i\frac{E_0^2\omega_c^2(3\omega^2-\omega_c^2)}{8\omega^3(\omega^2-\omega_c^2)^2}\sin2\omega t-i\left[(n_z+\tfrac{1}{2})\omega_c+\frac{E_0^2\omega_c^2}{4\omega^2(\omega^2-\omega_c^2)}\right]t\right]\ , \tag{8}$$

where $H_n(x)$ is a Hermite polynomial, and

$$\xi(t)=\frac{E_0\omega_c^2}{\omega(\omega^2-\omega_c^2)}\cos\omega t$$

is the classical solution of the forced harmonic oscillator, whose amplitude goes to zero in the vanishing magnetic field limit.

The wave functions $\psi_f^x(p_x,t)$ and $\psi_f^y(p_y,t)$ are momentum normalized while $\psi_f^z(p_z,t)$ is normalized to unity. The set of quantum numbers that specifies the final state is (p_x^f, p_y^f, n_z).

We see from Eq. (8) that $\xi(t)$ represents the main coupling between the motion in a magnetic field and the motion in a laser field. Since the classical solution for the momentum of a free electron in a laser field alone is $p_z^c(t)=(E_0/\omega)\cos\omega t$, the importance of this coupling to multiphoton processes, in the weak magnetic field or the weak laser field limit, is measured by

$$\frac{\xi(t)}{p_z^c(t)}=\frac{\omega_c^2}{\omega^2-\omega_c^2} .$$

This argument suggests that the usual perturbative approach[2-8] is valid if $\omega \gg \omega_c$ and the laser is weak, but it also suggests that changes have to be made if $\omega \sim \omega_c$, where the electronic motion in the uniform static magnetic field is resonantly coupled to the laser field. New evidence of this coupling has been found in a recent experiment on stimulated electromagnetic emission in the ionosphere.[17]

III. MULTIPHOTON DETACHMENT CROSS SECTIONS

Assume the field strengths and interaction time are such that the negative ion is only slightly perturbed by the interaction, and ignore the effect of the core on the detached electron. Reiss has shown that the S-matrix element for the detachment process is given by[16]

$$S_{fi}=-i\int_{-\infty}^{\infty}\langle\psi_f|V_f|\psi_i\rangle dt =i\int_{-\infty}^{\infty}\langle\psi_f|\tfrac{1}{2}p^2-\varepsilon_i|\psi_i\rangle dt . \quad (9)$$

Assuming the initial-state wave function can be represented by $\psi_i=\phi_i(\mathbf{p})\exp(-i\varepsilon_i t)$ and using the solution ψ_f given in Sec. II, we have

$$\langle\psi_f|\tfrac{1}{2}p^2-\varepsilon_i|\psi_i\rangle=\frac{1}{2\pi i}Q(t)\exp\{i[\varepsilon_f^x+(n_z+1/2)\omega_c+s'-\varepsilon_i]t\} , \quad (10)$$

where

$$s'=\frac{E_0^2}{4(\omega^2-\omega_c^2)} \quad (11)$$

is the magnetic-field-modified ponderomotive shift and

$$Q(t)=\frac{i\pi\omega_c^{-1/4}}{(2^{n_z}n_z!\sqrt{\pi})^{1/2}}\exp(ip_y^f\xi/\omega_c+iv'\sin 2\omega t)\int_{-\infty}^{\infty}(p_x^{f^2}+p_y^{f^2}+p_z^2-2\varepsilon_i)\phi_i(p_x^f,p_y^f,p_z)H_{n_z}((p_z-\xi)/\omega_c^{1/2})$$
$$\times\exp\left[-\frac{1}{2\omega_c}(p_z-\xi)^2+i\left[\frac{1}{\omega_c}p_y^f+\frac{E_0}{\omega^2-\omega_c^2}\sin\omega t\right](p_z-\xi)\right]dp_z , \quad (12)$$

where

$$v'=\frac{E_0^2(\omega^2+\omega_c^2)}{8\omega(\omega^2-\omega_c^2)^2} .$$

$Q(t)$, given by Eq. (12), is a periodic function of t with frequency ω, and therefore can be expanded as a superposition of photon-number states, i.e.,

$$Q(t)=\sum_{N=-\infty}^{\infty}S_{fi}^{(N)}e^{-iN\omega t} . \quad (13)$$

The S-matrix element S_{fi} and the N-photon detachment cross section are then given, respectively, by

$$S_{fi}=\sum_{N=-\infty}^{\infty}S_{fi}^{(N)}\delta(\varepsilon_f^x+(n_z+1/2)\omega_c+s'-\varepsilon_i-N\omega) , \quad (14)$$

and

$$\sigma^{(N)}=\frac{8\pi\omega}{cE_0^2}\sum_{n_z}\int W_{fi}^{(N)}dp_x^f dp_y^f , \quad (15)$$

where the transition rate $W_{fi}^{(N)}$ is given by

$$W_{fi}^{(N)}=(2\pi)^{-1}|S_{fi}^{(N)}|^2\delta(\varepsilon_f^x+(n_z+1/2)\omega_c+s'-\varepsilon_i-N\omega) . \quad (16)$$

Some general observations can be made at this point. For $\omega \gg \omega_c$, the threshold shift becomes what is normally expected, i.e., the sum of the ponderomotive potential $s=E_0^2/4\omega^2$ and the zero point energy in a magnetic field, $\omega_c/2$. But for smaller frequencies, especially for those near the cyclotron frequency ω_c, the threshold shift is changed by the magnetic field in a dramatic fashion. The magnetic-field-modified ponderomotive shift s' goes through a resonance at the cyclotron frequency ω_c and becomes negative when $\omega<\omega_c$. If this negative threshold shift is such that $-s'>|\varepsilon_i|+\omega+\omega_c/2$, which could be satisfied by using a strong laser at frequencies below the cyclotron frequency, the process of detachment with simultaneous induced photon or multiphoton emission becomes possible. For typical lasers having a frequency of the order of 10^{14} Hz, a magnetic field strength of the order of 10^4 T is required to "see" the region of the cyclotron resonance. The electron energy spectrum, especially as a function of the magnetic field, should be able to exhibit the effects of the magnetic-field-modified ponderomotive shift when such strong magnetic field strengths become experimentally achievable.

For ordinary magnetic field strengths of 1 T or smaller, the cyclotron resonances are in the microwave region or lower. When the electromagnetic fields are in this frequency region, the picture of a tilting electric field may be more convenient in explaining the detachment process, while whatever happens afterwards can be explained by the classical equations of motion.[18] This is one of the subjects we hope to address in the future.

IV. NONRESONANT, WEAK MAGNETIC FIELD LIMIT

We assume in this section the same conditions under which all the experimental and theoretical works so far on one-photon detachment in an external magnetic field have been carried out, i.e., $\omega_c \ll |\varepsilon_i|$ and $\omega_c \ll \omega$, so that $s' \to s$ and $v' \to v$, where s and v are defined below Eq. (6). To lowest order in ω_c, the final-state wave function, given by Eqs. (4)–(6) and (8), becomes

$$\psi_f(\mathbf{p},t) \approx \delta(p_x - p_x^f)\delta(p_y - p_y^f)\frac{\omega_c^{-1/4}}{(2^{n_z} n_z! \sqrt{\pi})^{1/2}} H_{n_z}(p_z/\omega_c^{1/2}) \times \exp\left[-\frac{1}{2\omega_c}p_z^2 - \frac{i}{\omega_c}p_y^f p_z - i[\varepsilon_f^x + (n_z+1/2)\omega_c]t\right]\exp\left[-i\left[\frac{E_0 p_z}{\omega^2}\sin\omega t + v\sin 2\omega t + st\right]\right] . \tag{17}$$

Recall that the solution for a free electron in a laser field (Volkov state[19]) is given by

$$\psi_V(\mathbf{p},t) = \delta(p_x - p_x^f)\delta(p_y - p_y^f)\delta(p_z - p_z^f)\exp(-ip^{f^2}t/2)\exp\left[-i\left[\frac{E_0 p_z}{\omega^2}\sin\omega t + v\sin 2\omega t + st\right]\right] . \tag{18}$$

We see immediately that Eq. (17) can be obtained simply by replacing the free-electron wave function in the Volkov state by the solution of a free electron in a static magnetic field. In other words, the effect of the laser and that of the magnetic field are separable in the weak-magnetic-field limit. We can disregard the magnetic field when evaluating the oscillator strengths, and take its effect into account by simply redistributing them among the proper set of final-state channels. In reaching this conclusion, we have also used the fact that a weak magnetic field has negligible effect on the ion at small distances.

Equation (12) can be simplified in the weak-magnetic-field limit by noticing that the slowly varying part in the integral can be replaced by its value at the stationary phase point $p_z = ip_y^f$. We obtain

$$Q(t) \approx \frac{i^{n_z+1}\sqrt{2}\pi^{3/2}\omega_c^{1/4}}{(2^{n_z} n_z!\sqrt{\pi})^{1/2}}(p_x^{f^2} - 2\varepsilon_i)\psi_i(p_x^f, p_y^f, ip_y^f)\exp\left[-\frac{1}{2\omega_c}p_y^{f^2}\right]H_{n_z}(p_y^f/\omega_c^{1/2})\exp\left[-\frac{E_0 p_y^f}{\omega^2}\sin\omega t + iv\sin 2\omega t\right] . \tag{19}$$

Its Fourier expansion gives

$$S_{fi}^{(N)} \approx \frac{(-1)^N i^{n_z+1}\sqrt{2}\pi^{3/2}\omega_c^{1/4}}{(2^{n_z} n_z!\sqrt{\pi})^{1/2}}(p_x^{f^2} - 2\varepsilon_i)\psi_i(p_x^f, p_y^f, ip_y^f)\exp\left[-\frac{1}{2\omega_c}p_y^{f^2}\right]H_{n_z}(p_y^f/\omega_c^{1/2})J_N\left[i\frac{E_0 p_y^f}{\omega^2}, -v\right] , \tag{20}$$

where $J_n(x,y)$ is the generalized Bessel function.[20] From Eqs. (15), (16), and (20), we get the multiphoton detachment cross section in a weak magnetic field:

$$\sigma^{(N)} = \frac{16\pi^3\omega\omega_c^{1/2}}{cE_0^2}\sum_{n_z=0}^{n_{\max}}\frac{(p_{n_z}^2 - 2\varepsilon_i)^2}{2^{n_z} n_z!\sqrt{\pi}p_{n_z}}\int_{-\infty}^{\infty}\left|\psi_i(p_{n_z}, p_y^f, ip_y^f)J_N\left[i\frac{E_0 p_y^f}{\omega^2}, -v\right]\right|^2 \exp(-p_y^{f^2}/\omega_c)H_{n_z}^2(p_y^f/\omega_c^{1/2})dp_y^f , \tag{21}$$

where

$$p_n = \sqrt{2\omega_c(\beta - n - 1/2)} ,$$

$$n_{\max} = [\beta - 1/2] ,$$

and where $[x]$ represents the largest integer that is smaller than or equal to x, and we have defined the scaled energy β as

$$\beta = (\varepsilon_i + N\omega - s)/\omega_c .$$

In a weak magnetic field, the most interesting region is near the detachment threshold, where the effect of the magnetic field is significant. To obtain some meaningful numerical results, let us consider the multiphoton detachment of an s-shell electron in the energy range of the first few Landau levels where $p_{n_z}^2 \sim \omega_c \ll 2|\varepsilon_i|$. Since the contribution to the integral in Eq. (21) comes primarily from the region $|p_y^f| \sim \omega_c^{1/2} \sim 0$, we have $\psi_i(p_x^f, p_y^f, ip_y^f) \approx \text{const} \equiv A/\sqrt{4\pi}$ and $p_{n_z}^2 - 2\varepsilon_i \approx -2\varepsilon_i$. Using the small argument expansion of the generalized Bessel function,[20] Eq. (21) gives[21]

$$\sigma^{(N)} = \begin{cases} 4\varepsilon_i^2\pi^2|A|^2 \dfrac{\omega_c^2}{c\omega^3}[J_{(N-1)/2}(v)+J_{(N+1)/2}(v)]^2 \displaystyle\sum_{n_z=0}^{n_{\max}} \frac{(n_z+1/2)}{p_{n_z}} \quad (N \text{ is odd}) & (22) \\ \dfrac{16\varepsilon_i^2\pi^2|A|^2\omega\omega_c}{cE_0^2} J_{N/2}^2(v) \displaystyle\sum_{n_z=0}^{n_{\max}} \frac{1}{p_{n_z}} \quad (N \text{ is even}) , & (23) \end{cases}$$

or when written in terms of the field-free detachment cross sections[22] near the detachment threshold,

$$\frac{\sigma^{(N)}}{\sigma^{(N)}(B=0)} = \begin{cases} \dfrac{3}{4\beta^{3/2}} \displaystyle\sum_{n_z=0}^{n_{\max}} \frac{n_z+1/2}{(\beta-n_z-1/2)^{1/2}} \quad (N \text{ is odd}) & (24) \\ \dfrac{1}{2\beta^{1/2}} \displaystyle\sum_{n_z=0}^{n_{\max}} \frac{1}{(\beta-n_z-1/2)^{1/2}} \quad (N \text{ is even}) & (25) \end{cases} .$$

These two modulation factors are shown in Figs. 1 and 2. They depend on the magnetic field and laser intensity only through the scaled energy β. As we show in the Appendix, Eq. (25) applies also to the case of parallel magnetic field and laser polarization. In fact, its applicability is expected to be independent of the direction of the magnetic field. Figure 3 shows the modulation factor, given by Eq. (A5), for multiphoton detachment by an odd number of photons for the case of parallel magnetic and laser fields.

Due to the Wigner threshold law, the N-photon detachment cross section for an s-electron near threshold is dominated by the p-wave contribution for odd N and by the s-wave contribution for even N. The fact that we have the same modulation factor for all odd N processes and for all even N processes implies that the effect of a weak magnetic field is completely specified by the final state, independent of the way the electron gets there. It is therefore our expectation that the idea of the frame transformation[8-10] may be applied to multiphoton detachment in a weak magnetic field to take into account the effect of the atomic core.

It is straightforward to show that for one-photon detachment of a 1s-type electron whose ground-state wave function can be written in the form of Eq. (26), Eq. (24) applies to the whole spectrum (rather than only near threshold) in the weak-laser and weak-magnetic-field limit.

V. MULTIPHOTON DETACHMENT OF H$^-$ BY A WEAK LASER FIELD

For a strong magnetic field, we can no longer separate the effect of the magnetic field from that of the laser. To

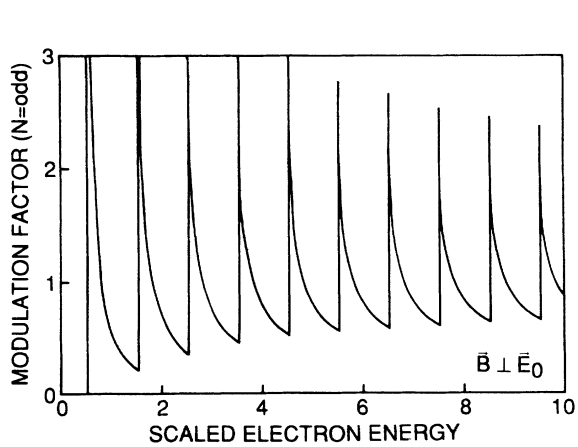

FIG. 1. Modulation factor for detachment by an odd number of photons vs scaled electron energy $\beta=(\varepsilon_i+N\omega-s)/\omega_c$ for the case $\mathbf{B}\perp\mathbf{E}_0$.

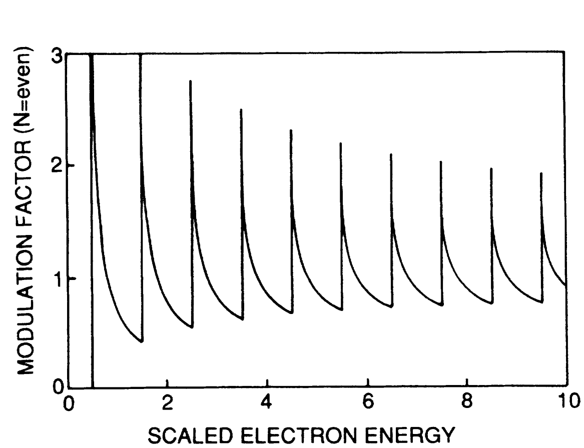

FIG. 2. Modulation factor for detachment by an even number of photons vs scaled electron energy $\beta=(\varepsilon_i+N\omega-s)/\omega_c$. $\mathbf{B}$ can be in any direction.

avoid complications introduced by the initial-state wave function, we use H^- as an example to discuss the effect of arbitrary magnetic field strength on the multiphoton detachment cross section. It is known that the wave function for the outer electron in the ground state of H^- can be well represented by a pure 1s-type orbital which, in momentum space, is given by[23]

$$\phi_i(\mathbf{p})=(A/\sqrt{4\pi})\frac{2|\varepsilon_i|}{p^2-2\varepsilon_i}, \tag{26}$$

where $\varepsilon_i=-0.027\,750\,9$ (a.u.) and $A=16.079$. Equation (12) can be integrated[21] exactly to obtain

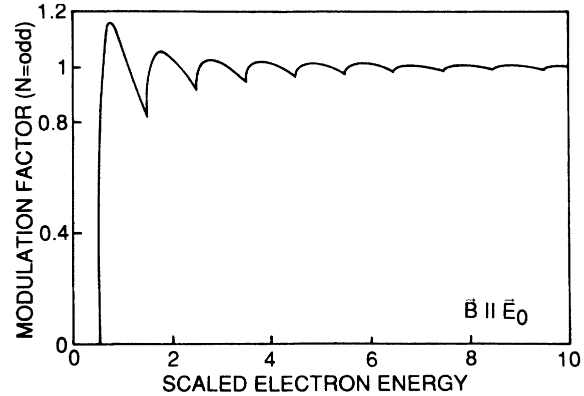

FIG. 3. Modulation factor for detachment by an odd number of photons vs scaled electron energy $\beta=(\varepsilon_i+N\omega-s)/\omega_c$ for the case $\mathbf{B}\|\mathbf{E}_0$.

$$Q(t)=\frac{i^{n_z+1}\sqrt{2}\pi A|\varepsilon_i|\omega_c^{1/4}}{(2^{n_z}n_z!\sqrt{\pi})^{1/2}}\exp\left[-\frac{1}{2\omega_c}p_y^{f2}-\frac{E_0^2\omega_c}{4(\omega^2-\omega_c^2)^2}\right]H_{n_z}\left[\left[p_y^f+\frac{E_0\omega_c}{\omega^2-\omega_c^2}\sin\omega t\right]\Big/\omega_c^{1/2}\right]$$
$$\times\exp\left[-\frac{E_0p_y^f}{\omega(\omega^2-\omega_c^2)^{1/2}}\sin(\omega t-i\delta_B)+i\frac{E_0^2}{8\omega(\omega^2-\omega_c^2)}\sin2(\omega t-i\delta_B)\right], \tag{27}$$

where δ_B is defined by $\tanh\delta_B=\omega_c/\omega$, or

$$e^{-\delta_B}=\left[\frac{\omega-\omega_c}{\omega+\omega_c}\right]^{1/2}.$$

The Fourier expansion of $Q(t)$ gives[14]

$$S_{fi}^{(N)}=(-1)^N\frac{i^{n_z+1}\sqrt{2}\pi A|\varepsilon_i|\omega_c^{1/4}}{(2^{n_z}n_z!\sqrt{\pi})^{1/2}}\exp\left[-\frac{1}{2\omega_c}p_y^{f2}-\frac{E_0^2\omega_c}{4(\omega^2-\omega_c^2)^2}\right]$$
$$\times\sum_{n=0}^{n_z}(-1)^n\begin{bmatrix}n_z\\n\end{bmatrix}\left[\frac{iE_0\omega_c^{1/2}}{\omega^2-\omega_c^2}\right]^n H_{n_z-n}(p_y^f/\omega_c^{1/2})$$
$$\times\sum_{k=0}^{n}(-1)^k\begin{bmatrix}n\\k\end{bmatrix}J_{N-n+2k}\left[\frac{iE_0p_y^f}{\omega(\omega^2-\omega_c^2)^{1/2}},-\frac{E_0^2}{8\omega(\omega^2-\omega_c^2)}\right]e^{-(N-n+2k)\delta_B}. \tag{28}$$

In the weak laser limit, the lowest-order terms are those with $k=0$ and $n\le N$, i.e.,

$$S_{fi}^{(N)}\xrightarrow{E_0\to0}(-1)^N\frac{i^{n_z+1}\sqrt{2}\pi A|\varepsilon_i|\omega_c^{1/4}}{(2^{n_z}n_z!\sqrt{\pi})^{1/2}}\exp\left[-\frac{1}{2\omega_c}p_y^{f2}\right]\sum_{n=0}^{N}(-1)^n\begin{bmatrix}n_z\\n\end{bmatrix}\left[\frac{iE_0\omega_c^{1/2}}{\omega^2-\omega_c^2}\right]^n$$
$$\times H_{n_z-n}(p_y^f/\omega_c^{1/2})J_{N-n}\left[\frac{iE_0p_y^f}{\omega(\omega^2-\omega_c^2)^{1/2}},-\frac{E_0^2}{8\omega(\omega^2-\omega_c^2)}\right]e^{-(N-n)\delta_B}, \tag{29}$$

where the generalized Bessel function can be replaced by its small argument expansion.[20] For one-photon detachment by a weak laser, we get

$$S_{fi}^{(1)}=i^{n_z}\frac{\pi A|\varepsilon_i|\omega_c^{1/4}E_0}{\sqrt{2}\omega(\omega+\omega_c)}\frac{1}{(2^{n_z}n_z!\sqrt{\pi})^{1/2}}\exp\left[-\frac{1}{2\omega_c}p_y^{f2}\right]\left[p_y^fH_{n_z}(p_y^f/\omega_c^{1/2})-\frac{2\omega_c^{1/2}\omega}{\omega-\omega_c}n_zH_{n_z-1}(p_y^f/\omega_c^{1/2})\right]. \tag{30}$$

From Eqs. (15) and (16), we obtain the one-photon detachment cross section by a weak laser field in a magnetic field of arbitrary strength:[21]

$$\sigma^{(1)}=\frac{4\varepsilon_i^2\pi^2|A|^2\omega_c^2}{c\omega(\omega+\omega_c)^2}\sum_{n_z=0}^{n_1}\left[\frac{\omega^2+\omega_c^2}{(\omega-\omega_c)^2}n_z+\frac{1}{2}\right]\Big/p_{n_z}\ , \tag{31}$$

where $n_1=[(\varepsilon_i+\omega)/\omega_c-1/2]$. This cross section is shown in Fig. 4 for $B=2\times10^8$ G. At this field strength, $\varepsilon_i+\omega_c$ falls between the $n_z=0$ and $n_z=1$ Landau thresholds. Comparing to the prediction of the weak field theory, i.e., the result obtained by multiplying the ratio in Eq. (24) by the field-free cross section,[14,16] the photon detachment cross section shows a broadening of the $n_z=1$ and higher Landau resonances and suppression of the $n_z=0$ Landau resonance.

The one-photon detachment cross section given by Eq. (31) is not resonant at the cyclotron frequency, since the only channel that could possibly be open at $\omega=\omega_c$ is $n_z=0$, regardless of the magnetic field strength. In fact, by using the small argument expansion of $J_{N-n}(x,y)$ given by Reiss,[20] we see immediately that the singularity at $\omega=\omega_c$ in Eq. (29) comes solely from the term $[iE_0\omega_c^{1/2}/(\omega^2-\omega_c^2)]^n$. When $\varepsilon_i+N\omega_c$ falls below the $n_z=1$ Landau threshold, the only channel that could be open at $\omega=\omega_c$ is $n_z=0$. In this case the $n=0$ term, which is nonsingular, is the only one contributing to the S matrix at the cyclotron frequency. Therefore, in the weak laser limit, the N-photon detachment cross section in general will not be resonant at the cyclotron frequency if $\varepsilon_i+N\omega_c$ is below the $n_z=1$ Landau threshold.

For more-than-one-photon detachment processes, it is possible to have $n_z=1$ up to $n_z=N-1$ Landau channels open at $\omega=\omega_c$, provided the magnetic field is strong enough so that

$$N\omega_c\geq3\omega_c/2+|\varepsilon_i|\ , \tag{32}$$

or $B\geq2|\varepsilon_i|/[(2N-3)\alpha]$. For H^-, this requires $B\geq1.305\times10^8$ G for two-photon detachment. When these conditions are satisfied, the cyclotron resonances are present explicitly in our free-electron picture. The smaller the binding energy and greater the N, the smaller the magnetic field that is required to see these resonances, which are due solely to the magnetic-field-induced electron-photon interaction.

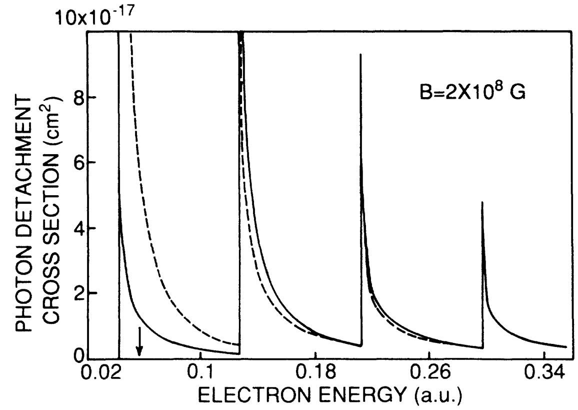

FIG. 4. One-photon detachment cross section (cm^2) of H^- in a magnetic field $B=2\times10^8$ G vs $(\varepsilon_i+\omega)$ (a.u.). Solid curve: Result of the strong field theory, Eq. (31). Dashed curve: Result of the weak field theory, Eq. (24), for $N=1$. The arrow indicates the position of $(\varepsilon_i+\omega_c)$.

For two-photon detachment by a weak laser, we get

$$S_{fi}^{(2)}=\frac{i^{n_z+1}\sqrt{2}\pi A|\varepsilon_i|\omega_c^{1/4}E_0^2}{16\omega^2(\omega+\omega_c)^2}\frac{1}{(2^{n_z}n_z!\sqrt{\pi})^{1/2}}\times\exp\left[-\frac{1}{2\omega_c}p_y^{f^2}\right]\left[-(\omega+2p_y^{f^2})H_{n_z}(p_y^f/\omega_c^{1/2})+\frac{16\omega\omega_c^{1/2}p_y^f}{(\omega-\omega_c)}n_zH_{n_z-1}(p_y^f/\omega_c^{1/2})-\frac{8\omega^2\omega_c}{(\omega-\omega_c)^2}n_z(n_z-1)H_{n_z-2}(p_y^f/\omega_c^{1/2})\right]\ , \tag{33}$$

from which we obtain the two-photon detachment cross section of H^- by a weak laser in a magnetic field of arbitrary strength:[21]

$$\sigma^{(2)}=\frac{\varepsilon_i^2\pi^2|A|^2\omega_cE_0^2}{16c\omega^3(\omega+\omega_c)^4}\sum_{n_z=0}^{n_2}[a_{n_z}^2(\omega)+4a_{n_z}(\omega)(n_z+1/2)\omega_c-a_{n_z}(\omega)b(\omega)n_z\omega_c+b^2(\omega)n_z(n_z-1/2)\omega_c^2/2-3b(\omega)n_z^2\omega_c^2+(6n_z^2+6n_z+3)\omega_c^2]/p_{n_z}\ , \tag{34}$$

where we have defined $n_2=[(\varepsilon_i+2\omega)/\omega_c-1/2]$, and

$$a(\omega)=\omega-\frac{4\omega^2\omega_c}{(\omega-\omega_c)^2}n_z\ ,$$

$$b(\omega)=8\omega(\omega-2\omega_c)/(\omega-\omega_c)^2\ .$$

We define here the generalized N-photon detachment cross section, which is independent of the laser intensity, as

$$\sigma_g^{(N)}=\frac{\sigma^{(N)}}{I^{N-1}}=\left[\frac{2^{N-1}}{E_0^{2N-2}}\right]\sigma^{(N)}\left[\frac{a_0^2}{I_0^{N-1}}\right]\ \text{in cm}^{2N}/\text{W}^{N-1}\ , \tag{35}$$

where $I = cE_0^2/8\pi$ is the laser intensity, a_0 is the Bohr radius, and $I_0 = ce^2/4\pi a_0^4 = 7.019\times10^{16}$ W/cm^2. Figures 5 and 6 show the generalized two-photon detachment cross section at two different magnetic field strengths. For $B < 1.305\times10^8$ G, there is no cyclotron resonance, but again the Landau resonances below $\varepsilon_i + 2\omega_c$ are suppressed, while the Landau resonances above it are broadened. For $B > 1.305\times10^8$ G, we indeed have the cyclotron resonance, which is so large that it dominates the whole spectrum.

Unlike the Landau resonances, which are basically due to the densities of final states, the cyclotron resonance is purely a dynamical effect that comes from the magnetic-field-induced photon absorption. It would not appear in any type of perturbative calculation that does not take into account the infinite-range electron-photon coupling induced by the magnetic field. To have a better understanding of this point, we can write the condition, Eq. (32), as $(N-1)\omega_c \geq |\varepsilon_i| + \omega_c/2$. It tells us that in the weak laser limit, we observe the cyclotron resonance when at least one of the photons with frequency ω_c is absorbed above threshold. We can understand this condition by noticing that if none of the photons is absorbed above the threshold, they must be absorbed at small distances, where the magnetic field would modify the electron-photon interaction but would not cause the cyclotron resonance due to the presence of the atomic core. But if one of the photons is indeed absorbed above threshold at $\omega = \omega_c$, it is most likely to be absorbed at large distances where the electron motion in the magnetic field is resonantly coupled to the laser radiation.

This condition for the occurrence of the cyclotron resonance is only necessary in the weak laser and free-electron approximation. In a strong laser field, cyclotron resonances can come from virtual absorption and emission at large distances even if $(N-1)\omega < |\varepsilon_i| + \omega_c/2 + s'$; it can also come from the coupling of different n_z channels induced by the effect of the atomic core.

Even though the free-electron approximation does not adequately describe multiphoton processes when there is no external static field (in the case where intermediate or final s states are possible), we can expect it to work very well in the strong static field limit, since the detachment cross section will ultimately be dominated by the static-field-induced part.

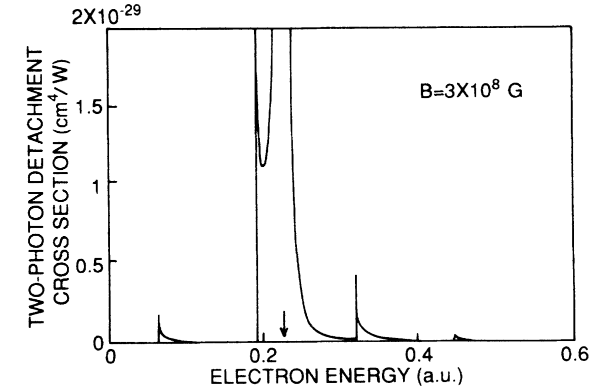

FIG. 6. The same as Fig. 5 for $B = 3\times10^8$ G.

Taking the limit $\omega_c \to 0$ in Eqs. (31) and (34), we get, as expected, results that are consistent with the conclusions of the previous section.

To demonstrate numerically the effect of the field-induced electron-photon coupling and the existence of cyclotron resonances, we have chosen the magnetic field strength of the order of 10^8 G in Figs. 4–6. In such a strong field, ω_c is of the same order of magnitude as the electron binding energy; therefore, a complete theory should include, in general, the effect of the magnetic field on the ground state. However, the inclusion of such an effect would not have altered the mechanism for the occurrence of cyclotron resonances, which is our major concern in this paper. Furthermore, in the case of very high N processes, the theory presented in this section is strictly valid in the region $0 < \omega_c \approx \omega \ll |\varepsilon_i|$, where the field-induced electron-photon coupling is important, while the effect of the magnetic field on the ground state in this region is not.

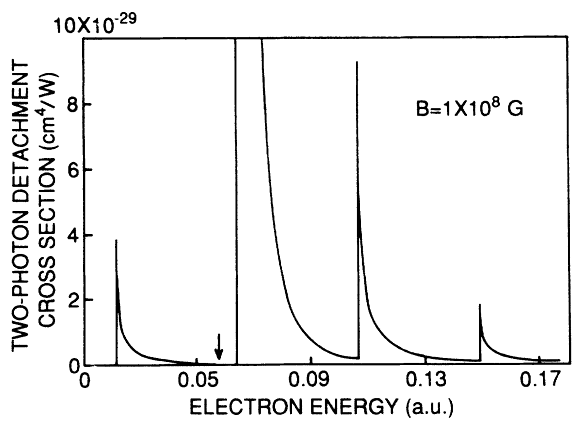

FIG. 5. Generalized two-photon detachment cross section (cm^4/W) of H^- in a magnetic field $B = 1\times10^8$ G vs $(\varepsilon_i + 2\omega)$ (a.u.). The arrow indicates the position of $(\varepsilon_i + 2\omega_c)$.

VI. CONCLUSIONS

Due to the fact that the electron can interact with photons in a static field, the electron-photon interaction generally has an infinite range for multiphoton processes in an external static field. As the static field strength increases, the field-induced electron-photon interaction becomes increasingly more important. In the case of a static uniform magnetic field, this interaction is especially important near the cyclotron frequency.

The effect of a weak magnetic field on the N-photon detachment cross section of an s electron can be characterized by two modulation factors, one for even N and one for odd N, confirming that the frame transformation technique should also work for multiphoton detachment in a weak magnetic field, regardless of the number of photons absorbed or the laser intensity (two conditions are implied by the term "weak," i.e., $\omega_c \ll \varepsilon_i$ and $\omega_c \ll \omega$). Cyclotron resonances, which come from the infinite-range character of the magnetic-field-induced electron-photon interaction, are shown to exist under certain conditions. They should be observable when a proper combination of magnetic field and laser frequency is used. Finally, the ponderomotive shift is modified by the magnetic field and becomes negative for laser frequencies smaller than the cyclotron frequency, which raises the interesting possibility of detachment with stimulated multiphoton emission for frequencies below the cyclotron frequency.

Strictly speaking, the validity of the dipole approximation we have used throughout this paper becomes questionable near the cyclotron resonance where the static-field-induced long-range electron-photon interaction plays an important role. The contribution of higher-order terms (such as the electric quadrupole and magnetic-dipole interactions) to the multiphoton detachment cross section in this region present an interesting problem which we hope to address in the future.

ACKNOWLEDGMENTS

I am deeply indebted to Professor Anthony F. Starace for suggesting this problem, reading the manuscript, and for constant support and encouragement. This work is supported in part by National Science Foundation Grant No. PHY-8908605, and by the Parker Foundation of the Department of Physics and Astronomy at the University of Nebraska–Lincoln.

APPENDIX: PARALLEL MAGNETIC FIELD AND LASER POLARIZATION

Let both fields be along the z direction. They can be described by

$$\mathbf{A}=(-By,0,(cE_0/\omega)\cos\omega t)\ . \tag{A1}$$

The wave function for a free electron in this field can be readily found:

$$\psi_f(\mathbf{p},t)=\delta(p_x-p_x^f)\frac{\omega_c^{-1/4}}{(2^{n_y}n_y!\sqrt{\pi})^{1/2}}H_{n_y}(p_y/\omega_c^{1/2})\exp\left[-\frac{1}{2\omega_c}p_y^2-\frac{i}{\omega_c}p_x^fp_y-i(n_y+\tfrac{1}{2})\omega_c t\right]$$
$$\times\delta(p_z-p_z^f)\exp\left\{-i\left[\frac{E_0p_z}{\omega^2}\sin\omega t+v\sin2\omega t+\left[\frac{p_z^{f^2}}{2}+s\right]t\right]\right\}, \tag{A2}$$

where v and s are defined below Eq. (6). Therefore in the parallel fields case, the effects of the two fields are completely separable at all field strengths. The electron-photon interaction remains short range, and there are no cyclotron resonances.

Proceeding similarly to the case of crossed fields discussed in the main text, we obtain

$$S_{fi}^{(N)}=(-1)^N\frac{i\pi\omega_c^{-1/4}}{(2^{n_z}n_z!\sqrt{\pi})^{1/2}}J_N\left[\frac{E_0p_z^f}{\omega^2},-v\right]\int_{-\infty}^{\infty}(p_x^{f^2}+p_y^2+p_z^{f^2}-2\varepsilon_i)\phi_i(p_x^f,p_y,p_z^f)$$
$$\times H_{n_y}(p_y/\omega_c^{1/2})\exp\left[-\frac{1}{2\omega_c}p_y^2+\frac{i}{\omega_c}p_x^fp_y\right]dp_y\ . \tag{A3}$$

Notice that the final state is now specified by the set of quantum numbers (p_x,n_y,p_z). The N-photon detachment cross sections are given by

$$\sigma^{(N)}=\frac{4\omega}{cE_0^2}\sum_{n_y}\int|S_{fi}^{(N)}|^2\delta(p_z^2/2+(n_y+1/2)\omega_c+s-\varepsilon_i-N\omega)dp_xdp_z\ . \tag{A4}$$

For near threshold multiphoton detachment of an s-electron in a weak magnetic field, we get[14]

$$\frac{\sigma^{(N)}}{\sigma^{(N)}(B=0)}=\begin{cases}\dfrac{3}{2\beta^{3/2}}\displaystyle\sum_{n_y=0}^{n_{\max}}(\beta-n_y-1/2)^{1/2}\quad(N\text{ is odd}) & \text{(A5)}\\[2ex] \dfrac{1}{2\beta^{1/2}}\displaystyle\sum_{n_y=0}^{n_{\max}}\frac{1}{(\beta-n_y-1/2)^{1/2}}\quad(N\text{ is even})\ , & \text{(A6)}\end{cases}$$

where $n_{\max}$ and β have been defined below Eq. (21). We see that the detachment cross sections for even numbers of photons are modulated by the same factor in both the parallel fields and the perpendicular fields cases. This is because near the detachment threshold, electrons detached by an even number of photons come out isotropically in an s state. We can therefore, expect the same modulation factor to apply for all directions of the magnetic field.

In the special case of H^-, with its initial state wave function given by Eq. (26), we get

$$\sigma_{fi}^{(N)}=\frac{16\varepsilon_i^2\pi^2|A|^2\omega\omega_c}{cE_0^2}\sum_{n_y=0}^{n_{\max}} J_N^2\left[\frac{E_0 p_{n_y}}{\omega^2},-v\right]\Big/p_{n_y} , \tag{A7}$$

which applies to all magnetic and laser field strengths under the assumption that the effect of the atomic core can be completely ignored.

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