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Collapse of ^{85}Rb Bose–Einstein condensate: mean-field analysis of the time delay

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Abstract

The collapse experiment of a ^{85}Rb Bose–Einstein Condensate [E.A. Donley, et al., Nature 412 (2001) 295] is studied by numerical simulation. We show that the time delay before collapsing observed in the experiment depends not only on the interaction strength parameters but also on the collective motion before being tuned to collapse. We also qualitatively discussed two necessary mechanisms for beyond mean-field theories on collapsing phenomena.

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In Bose–Einstein Condensation (BEC) community, it is well-known that a condensate with attractive interaction (characterized by negative scattering length a) is metastable [1]. When its particle number N exceeds a certain value, it would collapse. This was initially demonstrated in ^{7}Li [2], and has been studied within the framework of the mean-field theory [3]. On the other hand, with the successful realization of the Feshbach resonance [4] in ultracold gases [5–7], the constraint on the initial particle number of BEC was overcome, and an almost pure condensate can be prepared at the initial time. What is more important, by this technique, the interaction of the condensate can be dynamically tuned and thereby many intriguing phenomena were uncovered, most of which challenge current theories [7].

In this Letter we focus on one of the phenomena discovered by JILA group, namely, there is always a time delay between the switching (to a negative scattering length) and the time when the collapse actually occurs [7]. All though several authors have studied this problem [8,9] and obtained the dependence of the delayed time (T_{delay}) on the mean-field strength parameters which is similar to that of experiment [7], our analysis and numerical simulation show that T_{delay} not only depends on the parameters but also on the collective motion of the condensate before set to collapse. We also briefly discussed necessary ingredients

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that should be included in beyond-mean-field theories when one tries to fully understand the dynamical collapsing phenomena.

Our work is based on the mean-field theory (MFT). It postulates that the many-body ground state can be described by a single-particle state which is macroscopically occupied by N bosons [1]. At zero temperature, the non-condensed component can be safely ignored, and the condensate (macro) wave function follows the Gross–Pitaevskii equation (GPE) [1]:

$$i\hbar\frac{\partial}{\partial t}\Phi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + g|\Phi|^2\right]\Phi(\mathbf{r},t), \quad (1)$$

where $g = 4\pi\hbar^2 Na/m$ is a parameter that characterizes the strength of mean-field interaction. For the JILA experiment [7] under investigation here, the trapping potential $V(\rho,z)$ is a cylindric harmonic oscillator (HO) potential with frequencies: $\omega_\perp = 2\pi \times 17.5$ Hz, and $\omega_z = \epsilon\omega_\perp = 2\pi \times 6.8$ Hz (where ϵ is the aspect ratio). For convenience, in the following discussions, length and time are scaled by $L_0 = \sqrt{\hbar/(2m\omega_\perp)}$ and $\omega_\perp^{-1}$, respectively.

Following the experiment procedure, we begin with a stable GPE ground state with positive a_{init}. Then a is adiabatically ramped (AR) down to near zero. After that, a is switched to pre-determined negative value to trigger collapse. This tuning effect can be best understood in Gaussian ansatz picture [1,10], in which widths of the condensate shape are projected into the coordinates of an imaginary particle that moves in an effective potential:

$$U(\rho,z) = \rho^2 + \rho^{-2} + \frac{\epsilon^2 z^2 + z^{-2}}{2} + \frac{g'}{\rho^2 z}, \quad (2)$$

where $g' = 4\sqrt{2/\pi}\,Na$. When $a > 0$, this effective potential only has one minimum. As a dynamically tuned down, the minimum is continuously lowered and moved towards the origin, which makes the imaginary particle oscillate around every instantaneous position of the minimum. When $a < 0$ but larger than one critical value a_{cr} [13]: $|a| < |a_{\text{cr}}|$, the particle is bounded around the minimum by a barrier from an abyss centered at the origin. While when $a < a_{\text{cr}} < 0$, the barrier vanishes and the particle falls into the abyss, which corresponds to the collapsing of the condensate. Nevertheless, the particle will have to spend on *the way to the abyss* a finite time, which is just the physical reason for the phenomenon of the time delay before collapse happens. On the one hand, that finite time relies on the interaction strength parameters Na: the larger Na is, the more abrupt the edge of the abyss in the effective potential and thus the shorter of the T_{delay}. On the other hand, if the imaginary particle has a velocity towards the origin before a tuned to negative, or it is closer to the abyss, T_{delay} becomes shorter, and vice versa.

From this intuitive picture, it is reasonable to conclude that the time delay phenomenon not only depends on the parameters Na, but is also sensitive to the initial state of the condensate before setting to collapse. To quantitatively study this phenomenon, we numerically solved GPE (1) with the second-order implicit alternating-direction algorithm [11], always starting from a stable GPE ground state whose wave function is achieved with the imaginary time evolution method [12]. We first studied the dependence of T_{delay} on the parameters. To exclude the influence of the initial motion of the condensate, after the static ground state is achieved, we suddenly switched a to negative without AR procedure. For $|a| < |a_{\text{cr}}|$ the condensate is found to show collective oscillation. For $a < a_{\text{cr}} < 0$, the wave function eventually shrinks to a singularity centered at the origin.

At this stage, it is necessary to clarify how the collapse phenomenon is defined in this particle-number-conserved simulation. In the prescription of MFT, the shrinking of the wave function would lead to an infinite increase of the particle number density at the origin (unless extra atom-loss term(s) is(are) introduced into the GPE). In a numerical simulation, however, the finite space grids and time step limit this infinite growth, and one can only expect spikes of the central density whose maximum values are grid-size dependent. Hence, in a particle-conserving description, it is reasonable to take the emergence of this type of spike structure as the signal of a collapse. There are several possible schemes to determine the onset of a collapse more quantitatively. One candidate is to set a "trigger" in the central density, above which the collapse is considered to occur [8]. An alternative is to use the peak position of the first spike [9]. In this Letter a new criteria is chosen: we numerically differentiate the central density curve versus time and take the position of

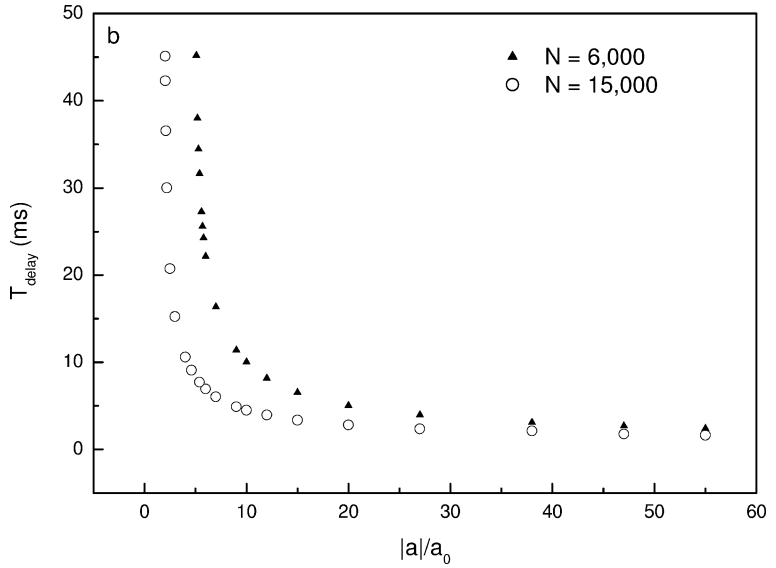

Fig. 1. The time delay before collapse versus $|a|$. $a_{\text{init}} = 0.7$ a.u. a_0 is the Bohr radius. See text for details.

the first spike in the derivative curve as the beginning of the collapsing. We believe that this scheme is more physically justified. Our criteria reflect the fact of the rapid growth of the central density in the collapsing regime. The first spike in the slope occurs before the first spike in density and provides a better indication that the central density is starting to rise abruptly and easier to be distinguished from the shape oscillation. For any time afterwards, the GPE is no longer applicable.

By this criteria for the *mean-field collapse*, the dependence of T_{delay} on a and N are determined. See Fig. 1. The profile of present curves agrees with previous experimental [7] and theoretical results [8,9], which in turn supports our simulation method. A quantitative comparison with experiment data can also be made. For $a_{\text{init}} = 7$ a.u., $a_{\text{collapse}} = -30$ a.u., $N_0 = 15\,000$, the experimental value of the delay is 3.75(5) ms, while our simulation gives 3.5 ms; for $a_{\text{init}} = 89$ a.u., $a_{\text{collapse}} = -15$ a.u., $N_0 = 6000$, the experimental value is $3 \times 3.75(5)$ ms, while our simulation result is 15.92 ms. We think those agreements are reasonable. Discrepancies could have come from any beyond-mean-field effects that might be present in the experiment and may also result from the condensate shape oscillation at the initial time as discussed in the following.

Next, we study the effect of the initial motion on T_{delay}. We begin with stable condensate with $a = 207$ a.u., and linearly ramp the bias field[1] until a reaches 0.7 a.u., within 200 ms and 800 ms, respectively. The condensate is left at this scattering length for another period of time (which we call it the relaxation time even though there is no damping mechanism in GPE) before we switch it to a negative scattering length and look at the time delay before collapse. Fig. 2(a), (b) depicts the time-variation of the mean square widths during AR for different ramping time, which illustrates the following points. First, it is difficult to achieve adiabatically. Even with a 800 ms ramp, we are exciting collective oscillations, which remains after the ramp has ended. Second, the faster the ramp, the greater the amplitude of the induced collective oscillation. Thus the condensate experiences shape oscillation during relaxation. This constitutes a continuous variation of the initial state before collapsing, leading to the changing of T_{delay}. Fig. 3 is the T_{delay} as a function of the relaxation time after AR procedure, which clearly shows the variation of T_{delay} *synchronizes* with the shape motion of the condensate. This effect has not been addressed in previous papers and

[1] The relation of a to the bias field we used is from [6].

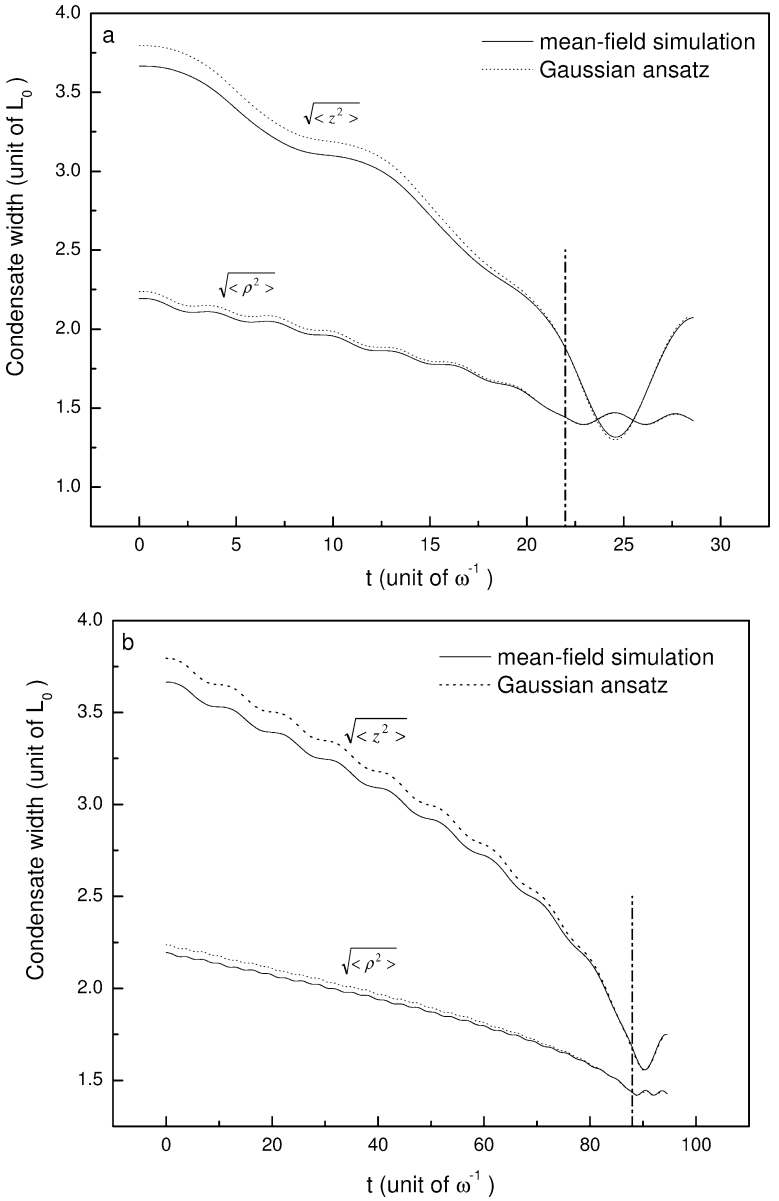

Fig. 2. Shape variation of the condensate during an adiabatic ramping and relaxation. Ramping time is: (a) 200 ms; (b) 800 ms. Relaxation time is 60 ms. The vertical dash-dot lines indicate when the ramping stops and the relaxation starts. The solid lines are the numerical simulation of GPE and the dashed lines are those of Gaussian ansatz.

should be observable in the experiment and may account for the discrepancy in T_{delay} between mean-field theory and experiment. On the other hand, since T_{delay} relates to the contraction speed of the condensate, this effect may be utilized as a tool to control the collapsing and exploding of the condensate, which complements the current parametric controlling method through a or N.

At the end of time delay process, it could be expected that the beyond-mean-field theory effects begin to play more and more important roles, which involves atom-loss mechanisms that disable the mean-field cri-

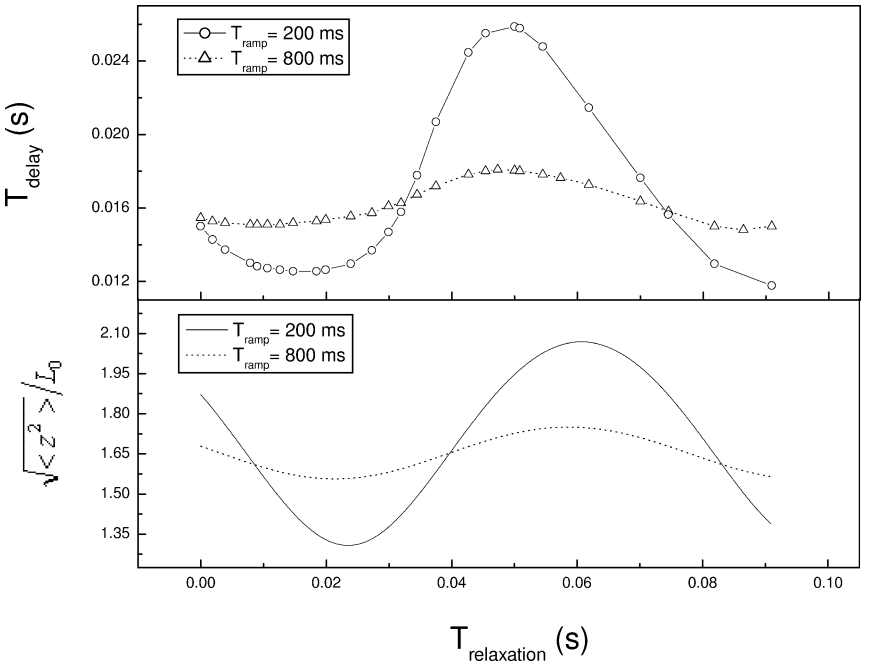

Fig. 3. The time delay versus the relaxation time. After the adiabatic ramping of the bias field, the condensate is relaxed to oscillate with $a_{\text{init}} = 0.7$ a.u. The solid and dashed lines are the mean squared axial width of the density profile. The symbols with connecting lines are those delays starting from different condensate state during relaxation. Solid lines represents results from 200 ms ramping while dashed lines for 800 ms ramping. The scattering length in collapse regime is -7 a.u. Particle number is 6000. The length scale in the figure is $L_0 = \sqrt{\hbar/(2m\omega_\perp)}$.

teria for collapse. When trying to precisely determine T_{delay} and fully understand collapsing and exploding phenomena, we think two following ingredients are essential. During collapsing, the condensate density can become sufficiently high that the mean-free-path of a hot atom can become shorter than the size of the condensate. This is especially true if the hot atom is created at an energy close to one of the resonances of two-body collisions (the resonance may either be a shape resonance or a Feshbach resonance). When this is the case, a hot atom will likely collide with remaining (cold) atoms to create other energetic atoms, which may again collide with others. This chain-collision mechanism has already been used to explain the enhanced trap-loss rate of a high density ^{87}Rb condensate [14], and we expect it to play a significant role in the understanding of collapsing and exploding of ^{85}Rb BEC. Another ingredient that is missing from the existing theories is the collision between the hot atom and molecules. From the experimental data [7], it is clear that the fraction of "missing atoms" can be quite large. If the hot atoms come primarily from the three-body recombination processes, this also implies that the number of molecules can be large. In this case, the ignorance of atom–molecule collision may again become difficult to justify.

In conclusion, time delay phenomenon in the dynamical collapse experiment for ^{85}Rb BEC is numerically studied with mean-field theory. We showed the delay relies not only on the interaction strength parameters but also on the initial collective motion of the condensate before tuning to collapse, the latter of which has not been discussed in literature. For future theory on a more realistic modeling of the collapsing and exploding dynamics, we qualitatively discussed necessary ingredients. We hope to address these issues and incorporate them into the study of collapse dynamics in the future.

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