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The effect of an optical plug on the tunable collapse

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Abstract

We propose using the large-detuned optical dipole force as an optical plug to control tunable collapse of Bose–Einstein condensation with attractive interactions. We show that the optical plug can increase the critical interaction strength while keeping ground state inter-atomic processes unchanged, and when the plug beam is strong enough, a new type of collapse can be induced. The phase diagram for these processes is presented with the mean-field analysis and numerical simulation.

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Ever since the success of gaseous Bose–Einstein condensation (BEC) experiments [1,2], far-off-resonance optical dipole force has been widely used in BEC research, such as helping to produce solitons [3], rotating condensate to create vortex lattice [4], making dipole confining [5] or periodical potentials [6], perturbing condensate for quasi-particle excitations [7], as well as splitting [8] and drilling [9] condensate. In this Letter, we propose a new usage of optical dipole force in BEC research, namely, using the far-off-resonance optical dipole force as an optical plug to control the collapse.

In trapped Bose–Einstein condensate with attractive interactions, when the interaction strength exceeds a certain value, the condensate rapidly contracts [10] and the particle density at its geometry center rises up sharply. This collapsing phenomenon has been observed in both experiments of ^{85}Rb BEC [11,12] and mean-field simulations [13]. The rapid growth of the density enhances loss processes that, after a certain time of delay, bring atoms out of the condensate and even kick them off the trap [12]. In other words, when collapse begins, a singularity appears at the center of the condensate, at which atoms effectively leak from the condensate. This picture is very analogous to the magnetic quadrupole trap, which holds a zero field point at the center, at which cold atoms escape from the trap via Majorana transition [14]. This *leaking at singularity* problem was solved by moving the *singularity* off the center to the surrounding so-called "death circle" [15], or by "plugging" the singularity with a blue-detuned laser beam [2].

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In this Letter, we suggest using a blue-detuned Gaussian beam passing through the axis of the condensate as an optical plug to control collapse, and study the effect of the optical plug on the tunable collapse. We focus on the static effect caused by this plug, such as the variation of the critical interaction strength. The optical plug affects collapse in two aspects: first, since the optical dipole force can be designed to be independent of any sublevel of atoms, and blue-detuned plug beam will repel atoms out of the center, employing such a plug will increase the critical interaction strength while keeping the magnetic sublevel structure of atoms unchanged due to far-off-resonance character, thus it will benefit investigations on collapsing dynamics. In contrast, magnetic methods that can change the critical interaction strength will also change the rates of inter-atomic processes, because those processes are sensitive to magnetic fields. Second, the combination of the optical plug with magnetic harmonic trap will build up a *doughnut* shape of the condensate in a certain parametric region, which qualitatively changes the showup of the collapse phenomenon, as shown in the following.

Our analysis is based on the mean-field theory. With the advancement of Feshbach resonance technique [11,16], the scattering length can be prepared to any value, which facilitates preparing a nearly pure condensate. Thus the Gross–Pitaevskii equation

$$i\frac{\partial}{\partial t}\Phi = \left[-\nabla^2 + V(\mathbf{r}) + \frac{8\pi Na}{L_0}|\Phi|^2\right]\Phi(\mathbf{r},t), \tag{1}$$

with the composite potential

$$\begin{aligned} V(\rho,z) &= V_{\text{trap}}(\rho,z) + V_{\text{ppt}}(\rho) \\ &= \frac{1}{4}\left(\rho^2 + \epsilon^2 z^2\right) + Ae^{-(\rho/w)^2}, \end{aligned} \tag{2}$$

can be safely applied in this context [17], where V_{trap} is the harmonic trap and V_{opt} represents the potential of the Gaussian beam that passes through the condensate axis and acts as the optical plug, with A the dipole force strength and w the beam width. In Eq. (1) we have scaled length by $L_0 = \sqrt{\hbar/2m\omega_\perp}$ and energy by $\hbar\omega_\perp$, where $\omega_\perp$ is the radial trapping frequency of the harmonic trap and ϵ the aspect ratio.

When the laser beam is absent, the attractive interaction strength ($g = 8\pi N|a|/L_0$) has a critical value g_{cr}, beyond which the condensate would collapse [12]. After the plug beam is switched on, since the blue-detuned laser drives atoms off the center against the attractive interaction and the trap, the critical constant should be shifted to higher values which depends on the parameters of the beam. These values are determined with numerical solution of Eq. (1). We use the finite difference scheme of ADI algorithm [18]. When $g < g_{\text{cr}}$, the ground state of Eq. (1) can be achieved with the imaginary time propagation method. If $g > g_{\text{cr}}$, the normalization constant during iteration diverges. We check every critical value for enough long numerical time (typically over $10\,000 \times 0.005$). When $A = 0$, our codes reproduce results reported in Ref. [19] satisfactorily.

The resulting critical values of g versus A are presented as curve a of Fig. 1. These points are obtained with imaginary time propagation starting from Gaussian function. As expected, exerted dipole force significantly lifts g_{cr} up, and g_{cr} monotonically increases with the dipole strength A. This can be understood since when the central barrier grows as the dipole strength tuned up, atoms have to gain more attractive interaction to surmount the barrier. Therefore, with this method, the allowed mean-field interaction strength can be made strong enough, which benefits the investigations on the collapsing dynamics after the beam is switched off.

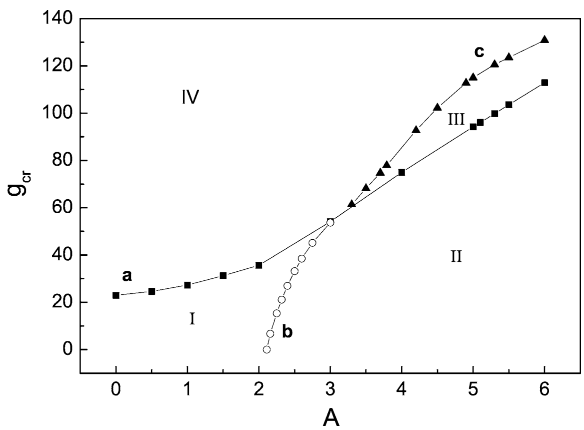

Fig. 1. The phase diagram for the effect of the optical plug on the collapse. Curve a (solid squares with line): the critical interaction strength for *central collapse*; curve b (empty circle with line): separatrix between Gaussian-like shape and doughnut shape of the condensate; curve c (solid triangles with line): the critical interaction strength for collapse of states of doughnut shape in parameter region III. The beam width is fixed to $w = \sqrt{3.0}$.

The result of curve a in Fig. 1 depends on the presumption that the ground state wave function is of Gaussian shape. But in fact, when A is large enough, the ground state wave function should show up a doughnut shape. The critical values of interaction strength for transition from Gaussian to doughnut shape vary with different A. We determined this separatrix in g_{cr}–A region by finding where the maximum density locates. One such curve is shown in Fig. 1 as curve b. Left to this separatrix is the region where the peak locates at the geometry center, and right to it is where the condensate density distribution develops a dip through the center and shows a doughnut shape.

The emergence of the doughnut structure significantly affects the collapse process. Consider a realistic situation. At first, the condensate is prepared into the doughnut shape with repulsive interaction. At the beginning time for collapsing, the scattering length is switched to a large negative value, and the condensate would contract due to the attractive interaction. However, if the laser beam strength A is large enough, it is so time-consuming for the condensate atoms to surmount the optical plug barrier, that condensate can collect sufficiently high density around every site on the torus before atoms tunneling through the barrier. In this case, collapse will happen first on the torus. This *torus collapse* is topologically different from those previously studied *central collapse*. For the latter, the condensate flows into the "black hole" and the singularity is a singular point. While for here presented case, the collapse happens on an torus and the whole condensate should shrink to a "super string" with infinitesimal width, if no loss mechanism is taken into account.

Since the Gaussian beam alters the trap, the effective trapping frequency along the torus is different from the global trapping parameters. Therefore, the critical value of the interaction strength for the torus collapse should be different from the central collapse. This inference is supported by our simulation. As an example, for $w = \sqrt{3}$, the critical values g_{cr} for the torus collapse are shown in Fig. 1 as curve c. This curve, together with that for central collapse (curve a) and the Gaussian-doughnut separatrix (curve b) divides the g_{cr}–A plane into four *phases*. In phase I the ground state is Gaussian and stable. In phase II, the numerical simulation produces the doughnut shape of the steady state. In phase III, the ground state is also of doughnut shape. However, states in this phase cannot be reached with the imaginary time evolution starting from Gaussian shape: those states will collapse into the central singularity instead of converging to doughnut shape. In our simulation, we have to prepare the condensate into the doughnut first and then switched the interaction strength to the predetermined value and evolve the condensate to convergence in imaginary time. States in phase IV is unconditional unstable: condensates in this parametric region will inevitably collapse.

The division of the parametric plane of g_{cr}–A into four phases causes important effects. First, the collapse not only relies on the parameters but also on the initial shape of the condensate. Second, the tuning route of the parameters (including the direction of tuning) also affects the collapse phenomenon. For example, tuning down the dipole strength A from phase II to phase III will lead the condensate to collapse, while the inverse tuning only results in shape oscillation. Third, the emergence of the torus collapse has physical significance. In reality, the collapse is stopped with the emerging of thermal atoms and molecules. In previous experiments [12] or numerical simulations [13], these products are burst mostly from the vicinity of the origin. In contrast, torus collapse will cause condensate lose components everywhere around the torus, which shall exhibit interesting dynamical phenomena.

Two remarks should be made at this stage. First, when the plug beam is too strong, it will be very difficult for the occurrence of the central collapse. Because if the plug is too strong, condensate has no time to gain enough density at the center before atoms slide into the toroidal dip surrounding the plug. As a consequence, atoms are amassed locally and exceeds the critical particle number for torus collapse. Therefore, curve a in Fig. 1 cannot be extended infinitely and phase III will be narrower with the increase of A. Second, additional caution ought to be given to the precision of g_{cr}. During simulation, if the starting point of the imaginary evolution is far from the target point in the parametric space, one may adiabatically increase g to g_{cr}. However, as g approaches g_{cr}, the energy surface becomes more and more flat. In this case, a static "collapsing" state can be quasi-stable: numerical fluctuations may not be able to boost it to the singularity in a short time, and one will have to wait for enough long time to see the collapse if it in principle should occur, which causes practical inconvenience. On the contrary, if one nonadiabatically tunes parameters, the condensate will gain extra artificial energy to surmount the energy barrier produced by the zero-point quantum motion, even though at this time g is still smaller than the real g_{cr}. Thus the boundary between collapse and metastability is blurred. We overcome this difficulty by combining nonadiabatical tuning with adiabatic tuning: we first use the former to search approximate g_{cr} and then use the latter to check it for sufficiently long numerical time. However, this methodology is not suitable to the part of curve a between phase II and phase III, where only nonadiabatic method is used instead, because when the interaction strength is adiabatically tuned up, states with parameters adjacent to those collapsing points will cross over curve a and transit to phase III.

In the above analysis, the width of the plug beam is fixed. The role of the beam width can be seen from the composite trap of the harmonic plus the Gaussian plug. For example, the section of the composite trap profile along one of the radial axes is demonstrated in Fig. 2(a). If w is very small, the plug acts just like a perturbing spike centered at the harmonic trap, and the collapse will not be changed much. With w increasing, the dip around the origin shifts out and lifts up, and the barrier around the origin begins to play role. Thus the critical interaction strength should increase. If the beam is too wide, the composite trap is nothing else but a shift-up harmonic-like trap, hence in this case g_{cr} shall not change much comparing with that without the plug. This analysis is reflected in Fig. 2(b) of g_{cr} versus w. For a certain value of A, the maximum of g_{cr} is achieved with moderate beam width.

In conclusion, we propose a novel method of using the large-detuned optical dipole force to control collapse. The blue-detuned optical plug dramatically changes the collapse in two ways. First, it increases the critical value of the interaction strength for the collapsing happened at the center. Second, the plug brings forth a new type of collapse, namely, the torus collapse, which occurs on the torus of the doughnut shape of the condensate. The laser power needed for this purpose is small. For example, for the ^{85}Rb BEC system at JILA, if 532 nm laser is used as the plug beam, only several µW is sufficient, which can be conveniently achieved and adjusted in the laboratory.

In this Letter, we present the phase diagram with mean-field analysis and numerical simulation. In practise, introducing Gaussian plug changes the trap structure, which shall induce not merely mean-field effects (such as shape oscillations) but also quantum processes, in particular, quantum tunneling is predicted to play active roles. Besides, the process of losing condensate atoms during implosion should also exhibit itself in manners different from what have been experimentally observed. We emphasize that optical plug not only is a complement and convenient control

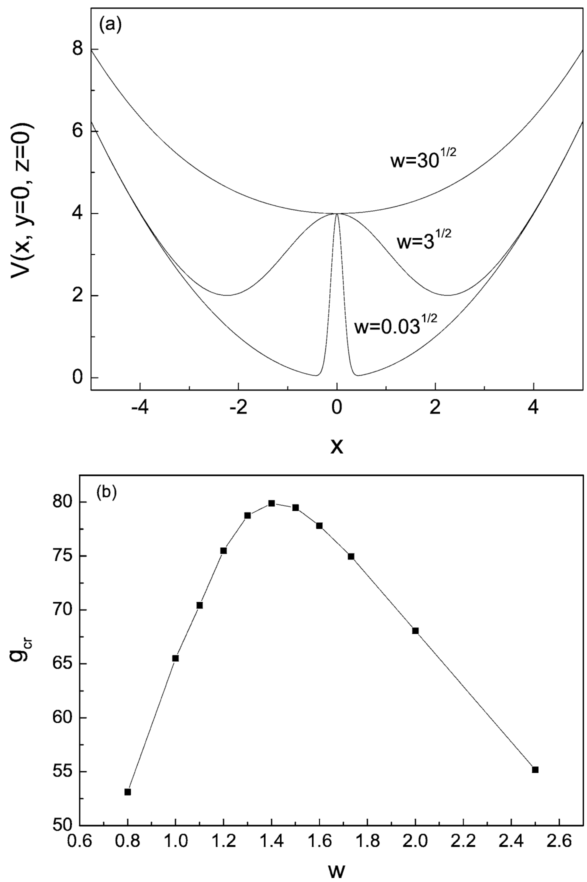

Fig. 2. Effect of the variation of the Gaussian plug beam width. (a) Section along the X axis of the composite potential $V(x, y, z)$ composed of the harmonic trap and the optical plug with different widths $w = \sqrt{30}, \sqrt{3}, \sqrt{0.03}$, from top to bottom, respectively. (b) The critical interaction strength g_{cr} versus the beam width w. The dipole force strength is fixed to $A = 4.0$.

methods for collapse, it will also induce many interesting phenomena which deserve further studies.

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